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LETTER TO THE EDITOR

The fractal nature of viscous fingering in porous media

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Abstract. Viscous fingering in porous media is an instability which occurs when a less viscous fluid displaces a more viscous one. An interface between the fluids is unstable against small perturbations and gives rise to a fingered configuration. In the oil industry viscous fingering can be a serious problem when displacing viscous oil by a more mobile fluid because it leads to poor recovery of the hydrocarbon. Recent work suggests an analogy between viscous fingering at an infinite viscosity ratio and diffusion-limited aggregation (DLA) and hence that the fingers may be fractal with a fractal dimension of around 1.7 (in two dimensions). This leaves unanswered the question of the nature of the fingered patterns at a finite viscosity ratio. To answer this a network model of the porous medium has been used. The rock is modelled as a lattice of capillary tubes of random radius through which miscible displacement occurs. At a high viscosity ratio and in the presence of a large amount of disorder the model reproduces DLA fingering patterns. The results of this model provide evidence that at a finite viscosity ratio the displaced area is compact with a surface fractal dimension between 1 and the DLA result of 1.7 with increasing viscosity ratio.

When a fluid is forced into a porous medium to displace another more viscous fluid the interface between the two fluids is unstable to small perturbations. As time proceeds the interface develops a highly complex fingered pattern. In the oil industry this can lead to a poor recovery of hydrocarbon as the displacing fluid bypasses the oil. This can have serious economic consequences and has therefore led to a number of experimental [1–3] and semi-empirical [4–6] methods to characterise the phenomenon. The first theoretical analysis by Saffmann and Taylor [7] was of the related problem of Hele-Shaw [8] flow. This model has subsequently received an intensive study both theoretical and experimental [9–11]. However, there are many differences between Hele-Shaw and porous medium flow. The macroscopic, continuum differential equations of flow are the same (Darcy's law [12, 13]) but in a porous medium flow occurs in a discrete, highly chaotic network of pores and pore throats. This adds a large noise component to the mean flow which is not present in Hele-Shaw flow. It is modelled in numerical simulations of porous medium flow by the inclusion of dispersion terms and larger scale variation in the permeability [14].

Recently this noisy element of porous medium flow has received some attention and has led to the development of stochastic models of the fingering phenomenon. In particular, an analogy [15] has been made between the diffusion-limited aggregation (DLA) model of Witten and Sander [16] and viscous fingering with a zero viscosity displacing fluid. Since then many attempts have been made to modify DLA to include surface tension effects [17, 18]. However, fewer attempts to allow for a finite viscosity

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ratio [19, 20] have been made, although other stochastic (non-random walk) models have been used [21, 22].

These stochastic models show that the viscous fingering patterns appear to be fractal and at an infinite viscosity ratio the fractal dimension is that of DLA (around 1.7). This has also been found for very high viscosity ratio displacements in Hele-Shaw cells [23, 24]. However, the models have been inconclusive about the fractal nature of viscous fingers at a finite viscosity ratio. The purpose of this letter is to examine this issue.

It is clear that the highly ramified structure of DLA cannot be sustained at a finite viscosity ratio. In order to supply a finger tip with fluid there must be a higher pressure at the base of the finger. This gives a transverse pressure gradient at the base and hence a lateral spreading there. From this it would seem plausible that the interior of a finger is compact (of dimension two) with a surface fractal region. For DLA the fingers are all surface with dimension 1.7. Hence we conjecture that the surface fractal dimension is a function of viscosity ratio varying between one (when the viscosities are matched) to 1.7 for an infinite viscosity ratio. Further the noise element of DLA may be reduced by requiring a site to be visited a certain number of times before growth is allowed there. It is known that this does not alter the fractal dimension of DLA [25]. Hence we also conjecture that the amount of noise present in the system does not affect the fractal dimension of the interface.

To test these conjectures we focus on viscous fingering on the pore scale. We use a network model similar to that used previously to examine pore scale fingering at an infinite viscosity ratio [26]. The model is a hexagonal network of cylindrical tubes of length L and variable radius r. The radii are taken uniformly from the interval $[1-\lambda, 1+\lambda]$ where λ is a variable disorder parameter. We use a hexagonal network rather than a square one because it is less prone to persistent grid effects at large distances [27]. Also this geometry is similar to close packed spheres in two dimensions. The tubes are considered to be long compared to their width so that the pressure drop associated with the nodes may be neglected. We also neglect mixing of the fluids at the nodes assuming that surface tension forces stabilise the interface without introducing a pressure drop. However, the displacement is assumed to be piston-like down each tube. The flow rate (Q_{ij}) down the tube connecting nodes i and j is given by Poiseuille's law

$$Q_{ij}=\frac{\pi r_{ij}^4(p_i-p_j)}{8[\eta_1 x_{ij}+\eta_2(L-x_{ij})]}=g_{ij}\Delta p_{ij}. \qquad (1)$$

Here p_i is the pressure at the ith node, η_1 is the viscosity of the injected fluid, η_2 is the viscosity of the displaced fluid and x_{ij} is the length of bond ij occupied by the injected fluid. The conductance of the bond g_{ij} is a function of time as the interfaces move. At each node there is a conservation of fluid

$$\sum_j Q_{ij}=0. \qquad (2)$$

To avoid the complications due to finite boundaries in a linear flood we use a radial configuration with injection of the less viscous fluid into the centre of a circular region (of radius r_0) initially occupied by the more viscous fluid. The central injection point and the circular perimeter are at constant pressure with a fixed (and arbitrary) pressure difference between the two. The viscosity (or mobility) ratio M is defined as

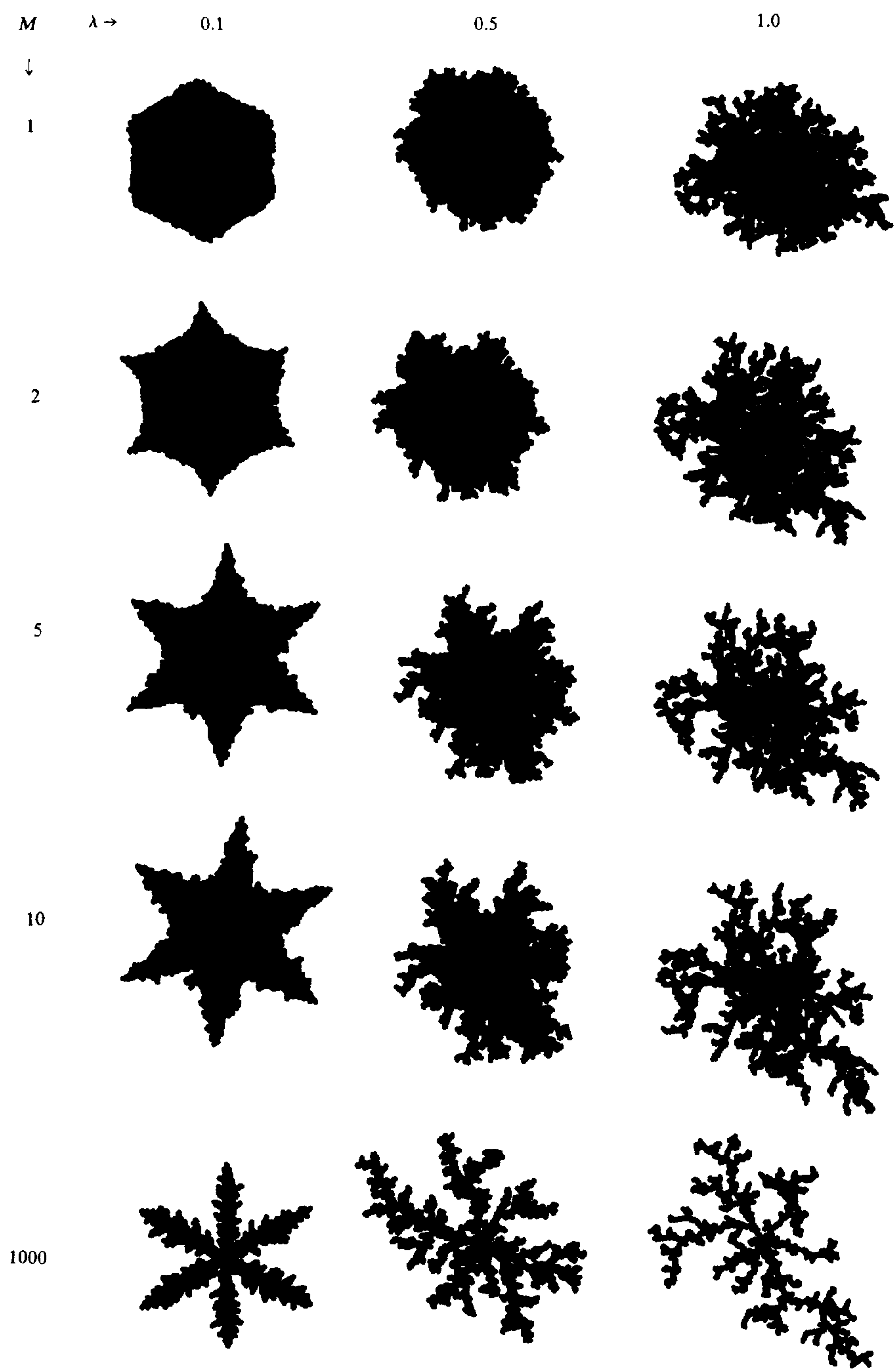

Figure 1. Computer simulations of viscous fingering at different values of the disorder parameter (λ) and viscosity ratio (M).

the ratio η_2/η_1. We solve for the pressures from (1) and (2) using successive over-relaxation

$$p_i^{n+1}=\frac{\alpha\sum_j g_{ij}p_j^n}{\sum_j g_{ij}}+(1-\alpha)p_i^n. \tag{3}$$

The over-relaxation parameter (α) was set at around 1.7. Once the pressure field has been established the fluxes in each tube are calculated from (1). A time step is calculated such that only one interface reaches a node and all the interfaces are updated accordingly. For the new configuration the pressure field is recalculated and the whole process is continued until the outer boundary is reached.

Figure 1 shows the results of the simulation for a variety of values of the disorder parameter λ and the viscosity ratio M. It can be seen that for small amounts of disorder there are very strong grid effects. Just how much disorder is required to overcome this is an interesting and as yet unanswered question. For the model considered it would appear that a value of λ between 0.1 and 0.5 is sufficient to destroy the grid effect. As the disorder and viscosity ratio increase the finger patterns look more like typical DLA simulations.

We divide the invaded sites into two types: interior sites, those for which all neighbouring nodes are also occupied by injected fluid; and surface sites, those which have one or more nearest neighbours occupied by the displaced fluid and which are on the interface (we exclude isolated, residual regions). To calculate the bulk fractal dimension (D_I) we counted the number of interior sites N_I inside a disc of radius r centred on the injection centre up to the radius of the whole region, r_0. This should scale as $N_\mathrm{I}\sim r^{D_\mathrm{I}}$. A log-log plot is shown in figure 2 for a typical case. In this and all other cases considered it is confirmed that the interior sites are compact, that is of dimension two. To calculate the surface fractal dimension (D_s) we used the two-point correlation function for the surface sites. We did this by counting the number of surface sites $n_i(r)$ in a shell around site i between radius r and $r+\Delta r$. The correlation function was then calculated from [28]

$$\rho(r)\sim\frac{1}{r}\sum_i n_i(r). \tag{4}$$

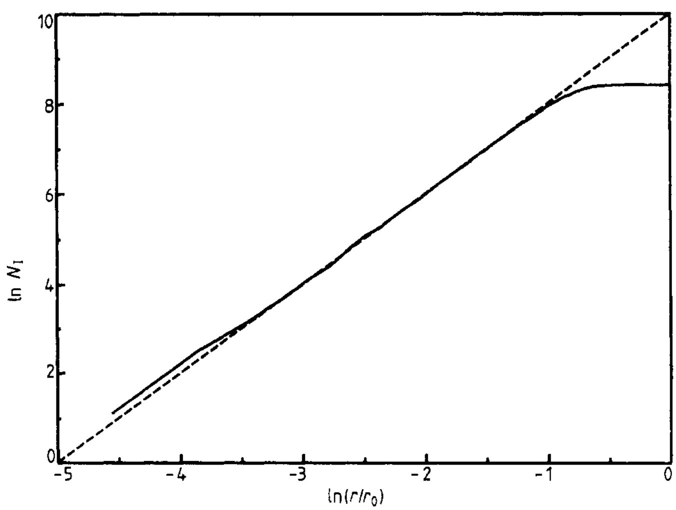

Figure 2. Plot of log (no of interior sites) against log (radius) to calculate the interior fractal dimension, with $M=5.0$, $\lambda=0.5$. The broken line is of gradient 2.

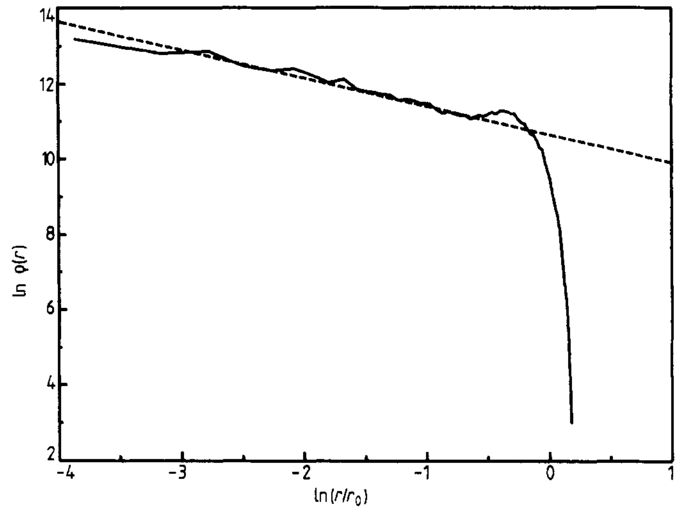

Figure 3. Plot of log (two-point correlation function) against log (radius) to calculate the surface fractal dimension. $M = 5.0$, $\lambda = 0.5$, $D_s \sim 1.25$ and the broken line is of gradient ~ -0.75.

We expect the correlation function to scale as $\rho(r) \sim r^{D_s - 2}$ in two dimensions. A typical log–log plot is shown in figure 3. For several realisations at both the same and different values of λ at a fixed viscosity ratio the results indicate that there is no dependence of the surface fractal dimension on the amount of disorder in the system. However, by performing the simulations at different values of the viscosity ratio it is shown that there is a systematic dependence of the surface fractal dimension on M. This is shown in figure 4. The error bars are derived from simulations at different and the same levels of disorder. The broken line represents the most simple polynomial fit to the results (equation (5)). It is not suggested that there is any theoretical justification behind this curve:

$$D_s = 1 + \frac{2}{3}\left(\frac{M-1}{M+1}\right)^2. \tag{5}$$

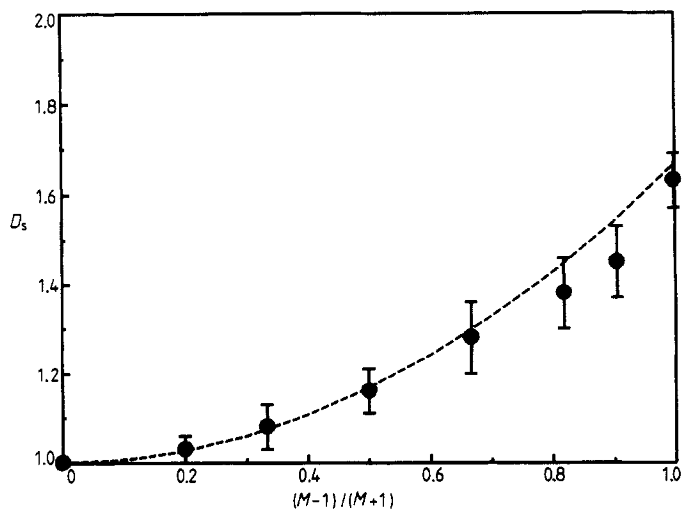

Figure 4. Surface fractal dimension as a function of viscosity ratio.

In conclusion we have used a network model of porous medium flow to provide support for the conjecture that viscous fingering is a surface fractal phenomenon. That is the interior of the fingers are compact. However, the interface region is fractal with dimension a continuous function of viscosity ratio between one (for unit viscosity ratio) to about 1.7 for the infinite viscosity ratio case analogous to diffusion-limited aggregation.

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